Hawking Radiation and Black Hole Evaporation

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Overview

- Classical black hole mechanics
- The quantum vacuum in Minkowski spacetime
- The Unruh effect
- Hawking radiation
- Black hole entropy
The Zeroth Law

- **Zeroth Law of Black Hole Mechanics**
  The surface gravity $\kappa$ is constant at the horizon of a stationary black hole

- **Zeroth Law of Thermodynamics**
  The temperature $T$ is constant throughout a system in thermal equilibrium
The First Law

✦ First Law of Black Hole Mechanics
If mass-energy $\delta M$ carrying angular momentum $\delta J$ and zero electric charge falls into a black hole, the change of surface area of the horizon $\delta A$ is given by

$$\delta M = \frac{\kappa}{8\pi} \delta A + \Omega \delta J$$

✦ First Law of Thermodynamics
The change of internal energy in a quasistatic process is the sum of the added heat and mechanical work done on the system:

$$\delta E = T\delta S + P\delta V$$
The Second Law

- **Second Law of Black Hole Mechanics**
  (Area theorem by HAWKING, 1971):
The surface area of the horizon can never decrease,

\[ \delta A \geq 0 \]

- **Second Law of Thermodynamics**
The total entropy of an isolated system can never decrease:

\[ \delta S \geq 0 \]
The Area Theorem

The total area of black hole horizons is monotonically increasing with time.
Black Hole Thermodynamics?

- Analogy to thermodynamics
  - Surface gravity $\kappa$ is a measure of temperature $T$
  - Horizon area $A$ is proportional to the entropy $S$

- Refutations (from the purely classical point of view):
  - The temperature of a black hole vanishes!
  - Entropy is dimensionless, the horizon area is a length squared!
  - The area is separately non-decreasing, whereas only the total thermodynamical entropy is non-decreasing!
  - Numerically, the black hole entropy is vastly larger than the entropy of a star from which the hole could have formed!
The Quantum Vacuum

- The definition of a particle (quantum) depends on the frame of reference.
- If the frames of two observers differ only by a Lorentz transformation, then they will agree about the particle content.
- If they have relative acceleration, then they will measure different particle numbers!
- The vacuum in Minkowski spacetime appears to be a thermal state when viewed by an accelerating observer (Davies, 1975, and Unruh, 1976).
Minkowski Spacetime

Geometric units: $c = 1, \ G = 1, \ k_B = 1$

\[ ds^2 = -dt^2 + dr^2 + r^2 d\Omega \]

- $r \to \infty \Rightarrow \text{spacelike infinity } I^0$
- $t \to +\infty \Rightarrow \text{future timelike infinity } I^+$
- $t \to -\infty \Rightarrow \text{past timelike infinity } I^-$

\[ ds^2 = -dudv + \frac{1}{4}(u - v)^2 d\Omega \]

- $u = t - r \to \infty \Rightarrow \text{future null infinity } \mathcal{I}^+$
- $v = t + r \to \infty \Rightarrow \text{past null infinity } \mathcal{I}^-$
Conformal Minkowski Spacetime

Compactify the manifold to include the boundaries $I^−, \mathcal{I}^−, I^0, \mathcal{I}^+ \text{ and } I^+$
QFT in a Nutshell

- Massless scalar quantum field $\phi$ in 1+1 Minkowski spacetime

- Field equation $\partial_\mu \partial^\mu \phi = -\phi_{,tt} + \phi_{,rr} = 0$
  $\Rightarrow$ plane-wave solutions $f^\pm_\omega = e^{-i\omega (t \mp r)} / \sqrt{2\omega}$

- Wave-packets following outgoing null rays onto $I^+$:
  $$\phi = \int d\omega \left[ a^+_\omega f^+_\omega + (a^+_\omega)^\dagger (f^+_\omega)^* \right]$$
  - $(a^+_\omega)^\dagger$ and $a^+_\omega$ are, respectively, creation and annihilation operators
  - Number operator for outgoing particles of frequency $\omega$ is $\mathcal{N}^\text{inert}_+ = (a^+_\omega)^\dagger a^+_\omega$
Inertial vs. Accelerated Observer

- **Basis modes in the inertial frame**
  \[ f^+_\omega = e^{-i\omega u \sqrt{2}} \quad f^-\omega = e^{-i\omega v \sqrt{2}} \]

- For a *Rindler* observer moving with const. acceleration \( a \), the null coordinates transform to \( u' \) and \( v' \)

- The transformation from the inertial to the accelerated frame is given by
  \[ ds^2 = -du'dv' = -e^{a(v'-u')}du'dv' \]

- The Rindler observer measures different time \( t' = \frac{1}{2}(u' + v') \) and length \( r' = \frac{1}{2}(v' - u') \) and has different basis modes
  \[ g^+_\omega = e^{-i\omega u' \sqrt{2}} \quad g^-\omega = e^{-i\omega v' \sqrt{2}} \]

\[
\phi = \int d\omega \left[ b^+_\omega \, g^+_\omega + (\, b^+_\omega \,)^\dagger (\, g^+_\omega \,)^* \right]
\]
The Unruh Effect

- Vacuum state $|0(I^-)\rangle$, $\forall \omega : a^-_\omega |0(I^-)\rangle = 0$
  - There are no in/outcoming particles seen by an inertial observer

- Number operator for outgoing modes in the accelerated frame is $\mathcal{N}^{+\text{Rindler}} = (b^{+\text{Rindler}})_\omega ^\dagger b^{+\text{Rindler}}_\omega$

- For the vacuum state $|0(I^-)\rangle$, the Rindler observer finds
  \[
  \langle 0(I^-)|\mathcal{N}^{+\text{Rindler}}|0(I^-)\rangle = \frac{1}{e^{2\pi \omega / a} - 1}
  \]
  - The Rindler observer beholds a flux of thermal radiation propagating toward $I^+$
  - The temperature of the vacuum is proportional to acceleration:
  \[
  T = \frac{\hbar a}{2\pi}
  \]
The Collapsing Star Spacetime

A black hole is a region in spacetime which is not in the past of of $I^+$
QFT in Curved Spacetime

- Use a prescribed background geometry with metric $g_{\mu\nu}$:

$$g^{\mu\nu}\partial_\mu\partial_\nu\phi = 0$$

- **Hawking (1973)** studied a scalar field $\phi$ propagating in the background spacetime around a star collapsing to a black hole

  - Towards $I^-$, the spacetime is asymptotically Minkowskian
  - Positive frequency modes are partially converted into negative frequency modes near the horizon (pair creation)
  - Whereas positive frequency modes scatter off the horizon, negative frequency modes are absorbed
  - Assuming the state $|0(I^-)\rangle$ in the remote past (the **Boulware state**), Hawking found that in the future of the collapse, there is a flux of thermal particles toward $I^+$
Virtual pair creation close to the horizon can result in a real particle escaping to $\mathcal{I}^+$ which carries away a fraction of the black hole mass.
Hawking Radiation

- A black hole produces thermal radiation with \( T = \frac{\hbar}{2\pi} \kappa \)
- The temperature is proportional to the surface gravity \( \kappa = \frac{1}{2} f'(R_H) = \hbar/8\pi M \), where \( f(r) = 1 - 2M/r \)
- The wavelength of Hawking radiation is \( \lambda_H = 8\pi^2 R_H \)
- \( T \) vanishes in the classical limit \( \hbar \to 0 \)
- \( a = \kappa/f^{1/2}(r) \) is the local acceleration of a stationary observer
- Hawking radiation is a consequence of the state near the horizon being the vacuum as viewed by free-fall observers
Black Hole Evaporation

- Negative frequency (energy) particle flux into $\mathcal{H}^+$ diminishes the black hole mass
- The rate of positive energy emission to $\mathcal{I}^+$ is $\propto M^{-2}$
- Black hole life time is of the order $M^3$
  - $(M/M_P)^3 T_P \sim 10^{-28} M^3$ [cgs]
  - A solar mass black hole ($M \sim 10^{33}$ g) emits thermal radiation of roughly $10^{-11}$ eV and lives about $10^{54}$ times the age of the universe
  - Only *primordial* mini black holes could be observed if they were around
Bekenstein-Hawking Entropy

There is entropy associated with a black hole horizon:

\[ S_{\text{BH}} = \frac{A_H}{4\hbar} \]

The unit of horizon area is \( L_P^2 \sim 10^{-66} \text{cm}^2 \)

For a stellar mass black hole, \( S_{\text{BH}} \sim 10^{78} \) which is about \( 10^{21} \) times the entropy of a collapsing star from which the hole could have formed

Generalised second law: The sum of BH entropy and the entropy outside black holes can never decrease

\[ \delta(S_{\text{outside}} + S_{\text{BH}}) \geq 0 \]
The Meaning of Black Hole Entropy

**HAWKING** (until last year): extra quantum uncertainty
- $S_{BH}$ corresponds to the maximum information lost in the BH
- Information about quantum states can not be restored

**PENROSE** (The Emperor’s New Mind): complementary quantum uncertainty
- Loss of information corresponds to shrinking of phase space volume as trajectories converge towards singularities
- This is counter-balanced by quantum measurements in which information and, thus, phase space volume is regained

**M-THEORY**: holographic principle and such
- Information is stored in the quantised horizon and eventually will be rejuvenated by BH evaporation
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